Reflection Problems in a Ferrite Stripline

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***Abstract*—In this paper, the transmission characteristics of a ferrite stripline are investigated in comparison with those of an ordinary dielectric substrate stripline. Also, both "short"-end and "open"-end reflection problems of a ferrite stripline are solved by means of the eigenmode expansion method; the results, which are confirmed by experiments, show that there is a large difference between the two reflection coefficients. For the short end, very little reflection occurs. Finally, as a practical application, a new type of EG mode isolator based on these results is proposed.**

I. Introduction

IN a ferrite stripline, we can induce the edge-guided mode (EG) propagating along one edge of stripline by applying a dc magnetic field perpendicular to the ground plane [1].

If the direction of the dc magnetic field is reversed, the EG mode will propagate along the other edge of the stripline. Therefore, the development of nonreciprocal microwave devices, such as isolator, circulator, and nonreciprocal phase shifter, can be facilitated through the application of this nonreciprocal transmission characteristic. Such EG mode devices are suitable for microwave integrated circuitry (MIC) because of their planar circuit structure (see Fig. 1).

In this paper, the transmission characteristics of a ferrite stripline are examined in comparison with those of an ordinary dielectric substrate stripline. Also, by means of the *eigen*mode expansion method, both the "short"-end and the "open"-end reflection problems of a ferrite stripline are solved; the results, also confirmed by experiments, show that there is a large difference between the two reflection coefficients.

Finally, as a practical application, the paper describes a new type of EG mode isolator based on these findings.

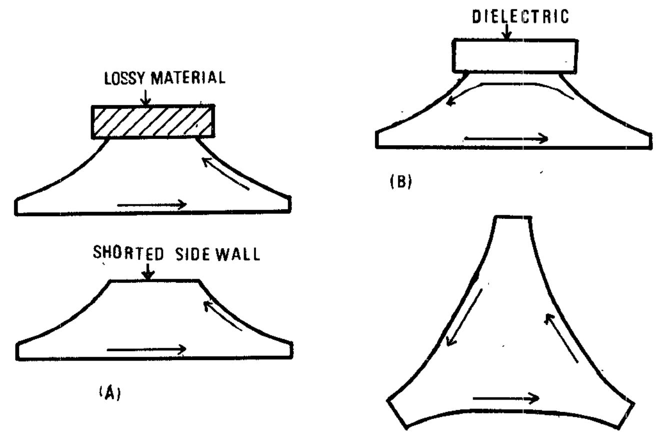

Fig. 1. EG mode devices. (a) Isolator. (b) Nonreciprocal phase shifter. (c) Circulator.

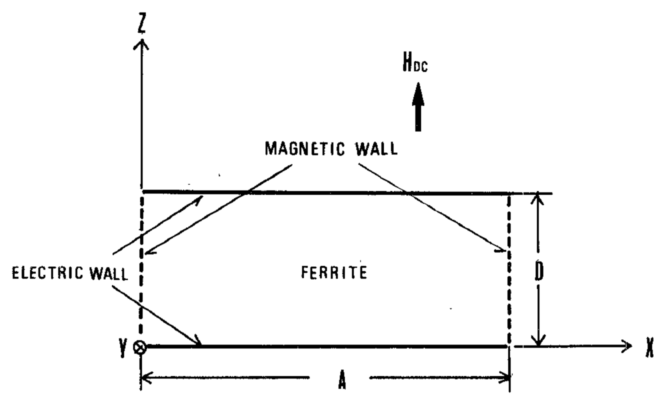

Fig. 2. Analytical model of a ferrite stripline.

II. Eigenmodes in a Ferrite Stripline

The EG mode will be able to exist in a stripline if a ferrite substrate is used. It is caused by the anisotropy of the magnetized ferrite. This mode propagates along the edge of the stripline, and hence it is a kind of surface wave, as it has been studied as a magnetostatic surface wave [2].

We will start by stating the general properties of eigenmodes in a ferrite stripline in comparison with the case of a dielectric substrate.

A. Eigenmode Function

Suppose the line structure is as shown in Fig. 2. Although there are the fringing fields at the edge of the stripline so that the open-boundary condition at the edge is inaccurate in strict meaning, we will neglect the fringing fields and assume the open-boundary condition for the convenience of analysis.

The thickness of the ferrite substrate is thin, and so the field will be assumed to have no variation with respect to the z-coordinate.

We have Maxwell's equation

$$\nabla \times \boldsymbol{E} = -j\omega[\mu]\boldsymbol{H} \quad (1)$$

$$\nabla \times \boldsymbol{H} = j\omega\varepsilon \boldsymbol{E}. \quad (2)$$

Therefore the only electric-field component E_z of the TE mode must obey the following wave equation:

$$(\nabla_t^2 + \omega^2\mu_{\text{eff}}\varepsilon)E_z = 0 \quad (3)$$

where ∇_t^2 is two-dimensional Laplacian and $\mu_{\text{eff}} = (\mu^2 - \kappa^2)/\mu$ is the effective permeability of the ferrite.

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Similarly, the magnetic field H_t must obey the same wave equation

$$(\nabla_t^2 + \omega^2\mu_{eff}\varepsilon)H_t = 0. \tag{4}$$

If the solution to (3) is found, the magnetic field can be solved by (1).

$$H_x = -\frac{1}{j\omega\mu_{eff}}\cdot\left[\frac{\partial E_z}{\partial y} - j\frac{\kappa}{\mu}\cdot\frac{\partial E_z}{\partial x}\right] \tag{5}$$

$$H_y = \frac{1}{j\omega\mu_{eff}}\cdot\left[\frac{\partial E_z}{\partial x} + j\frac{\kappa}{\mu}\cdot\frac{\partial E_z}{\partial y}\right]. \tag{6}$$

Let us suppose E_z in the form of separation of variables as follows:

$$E_z = \Psi(x)\cdot\exp(-\gamma y). \tag{7}$$

Then

$$\left(\frac{d^2}{dx^2} + \Gamma^2\right)\Psi(x) = 0 \tag{8}$$

where

$$\Gamma^2 = \gamma^2 + \omega^2\mu_{eff}\varepsilon \tag{9}$$

and the open-boundary conditions at $x = 0$ and a are

$$\left(\frac{d}{dx} - j\frac{\kappa}{\mu}\gamma\right)\Psi(x)\bigg|_{x=0,a} = 0. \tag{10}$$

The solution to the boundary value problem of (8) and (10) (i.e., eigenmode functions) will be easily obtained.

0th order:

$$\Psi_0(x) = \exp(-\alpha x) \tag{11}$$

where

$$\alpha = \frac{\kappa}{\mu}\omega\sqrt{\varepsilon\mu} \tag{12}$$

$$\gamma_0 = j\omega\sqrt{\mu\varepsilon}. \tag{13}$$

nth order:

$$\Psi_n(x) = \cos\Gamma_n x + j\frac{\kappa}{\mu}\cdot\frac{\gamma_n}{\Gamma_n}\cdot\sin\Gamma_n x \tag{14}$$

where

$$\Gamma_n = \frac{n\pi}{a} \tag{15}$$

$$\gamma_n = \sqrt{\Gamma_n^2 - \omega^2\mu_{eff}\varepsilon}. \tag{16}$$

Though higher order modes are TE mode, the 0th-order mode (EG mode) is a quasi-TEM mode.

Similarly, for the backward wave eigenmode functions are given as follows.

0th order:

$$\Phi_0(x) = \exp(\alpha x). \tag{17}$$

nth order:

$$\Phi_n(x) = \cos\Gamma_n x - j\frac{\kappa}{\mu}\cdot\frac{\gamma_n}{\Gamma_n}\cdot\sin\Gamma_n x. \tag{18}$$

If γ is a solution for the propagation constant, then $-\gamma$ will also be a solution; however, the mode function (i.e., field distribution) of the backward wave is not the same as that of the forward wave. For symmetrical edge-boundary conditions, the transmission is reciprocal.

However, when the line structure is not symmetrical, for example, one edge is shorted to the ground plane, $-\gamma$ will not be a solution. In such a situation, the nonreciprocal character is emphasized [3].

On the other hand, the magnetic fields corresponding to (11) and (18), are given from (5).

Forward Wave

0th order:

$$\eta_0(x) = \sqrt{\frac{\varepsilon}{\mu}}\cdot\exp(-\alpha x). \tag{19}$$

nth order:

$$\eta_n(x) = \frac{\gamma_n}{j\omega\mu}\cdot\cos\Gamma_n x - \frac{\kappa}{\mu}\frac{\omega\varepsilon}{\Gamma_n}\cdot\sin\Gamma_n x. \tag{20}$$

Backward Wave

0th order:

$$\xi_0(x) = -\sqrt{\frac{\varepsilon}{\mu}}\cdot\exp(\alpha x). \tag{21}$$

nth order:

$$\xi_n(x) = -\frac{\gamma_n}{j\omega\mu}\cdot\cos\Gamma_n x - \frac{\kappa}{\mu}\frac{\omega\varepsilon}{\Gamma_n}\cdot\sin\Gamma_n x. \tag{22}$$

B. Orthogonality of Eigenmodes in a Ferrite Stripline

In the general theory of waveguides, it is well known that "orthogonality" of eigenmodes exists, which is an important property [1]. Can orthogonality of eigenmodes exist for a ferrite stripline?

Introducing the off-diagonal element κ of the tensor permeability of a ferrite, we have the following relations between eigenmodes easily derived from (8) and (10):

$$(\gamma_m^* - \gamma_n)\int_0^a \Psi_m^*(x)\cdot\Psi_n(x)\,dx = j\frac{\kappa}{\mu}[\Psi_m^*(x)\cdot\Psi_n(x)]_0^a,$$
$$m, n = 0, 1, 2, \cdots, m \neq n. \tag{23}$$

Orthogonality of eigenmodes in mathematical expressions is equivalent to superpositionality of the power flow of eigenmodes in physical meaning. Even introducing κ, it will be shown that orthogonality is still valid, as follows.

Suppose that the mth and the nth mode simultaneously propagate, that is,

$$E_z = A_n\Psi_n(x)\exp(-\gamma_n y) + A_m\Psi_m(x)\exp(-\gamma_m y) \tag{24}$$

$$H_x = \frac{1}{j\omega\mu_{eff}}\left[A_n\left(\gamma_n\Psi_n(x) + j\frac{\kappa}{\mu}\cdot\Psi_n'(x)\right)\exp(-\gamma_n y) + A_m\left(\gamma_m\Psi_m(x) + j\frac{\kappa}{\mu}\Psi_m'(x)\right)\exp(-\gamma_m y)\right]. \tag{25}$$

TABLE I
THE PROPERTIES OF THE EIGENMODES

Dielectic substrate	Ferrite substrate
The eigen modes are a real function, that is, there is no phase difference in the cross section.	The eigen modes are a complex value function, except for the 0-th mode when ferrite is assumed to be lossless.
The ratio of Ez to Hx, wave impedance for each modes, are constant in the cross section.	The ratio of Ez to Hx are not constant, except for the 0-th mode.
The mode functions of forward wave are the same as that of backward wave.	The mode functions of forward wave are not the same as that of backward wave.
The power flow of evanescent higher modes is 0, at every points in the cross section.	The integrand of power flow across the cross section, for evanescent modes is 0, however, the power flow is not 0 at every points.
The superpositionality of power flow exists, even if dielectric is lossy.	The superpositionality of power flow exists, only if ferrite is lossless.

For this case, the cross term of the mth mode and the nth mode in power flow $P_{n,m}$ vanishes (see Appendix 1)

$$P_{n,m} = \mathrm{Re}\left\{\int_0^a \left[A_n^* A_m \Psi_n^*(x)\left(\gamma_m \Psi_m(x) + j\frac{\kappa}{\mu}\Psi_m'(x)\right) \cdot \exp\left[-(\gamma_n^* + \gamma_m)y\right] + A_m^* A_n \Psi_m^*(x) \cdot \left(\gamma_n \Psi_n(x) + j\frac{\kappa}{\mu}\Psi_n'(x)\right) \cdot \exp\left[-(\gamma_m^* + \gamma_n)y\right]\right] dx/j\omega\mu_{\mathrm{eff}}\right\} = 0. \quad (26)$$

When manipulating (26), we used the relation (23). On the contrary, if $P_{n,m} = 0$, (23) will be necessary. So we can call (23) the orthogonal relation between the mth mode and the nth mode.

But (26) can be obtained only when the ferrite is assumed lossless. That is, (26) does not stand when the ferrite is lossy.

For the case of a dielectric substrate, even if the substrate is lossy, the superpositionality of the power flow will be valid. It may be one of the differences between them. For this reason, the reflection problems are complicated in a ferrite stripline.

C. The Character of Eigenmodes

We determine here the character of the eigenmodes in a ferrite stripline, comparing the results to the case of a dielectric substrate (see Table I).

As previously stated, the κ term of the tensor permeability originates certain different characters. Moreover, it is important to verify whether the eigenmode sets $\{\Psi_n(x)\}$ and $\{\eta_n(x)\}$ obtained here are complete or not, as it is well known to be complete for the eigenmode sets in case of a dielectric substrate [4], because this property permits the use of an eigenmode expansion method for the general analysis involving reflection problems. Although the completeness of our sets have not been proved analytically yet, it is ascertained numerically that an eigenmode expansion method is useful for the reflection problems.

The reflection problem is a very fundamental one in a waveguide. When the mode functions of the forward and backward waves are the same in the cross section, the problem is easy. For instance, when the termination is either short or open, the voltage reflection coefficient is -1 or 1, respectively. However, in the case of the EG mode transmission line, the mode functions of the forward and backward waves are different as mentioned before, and so at the termination there must be mode conversions. We will discuss this problem in the following section.

III. AN ANALYSIS OF REFLECTION PROBLEMS

Reflection problems are analyzed by means of an eigenmode expansion method. Losses of ferrite inevitable in practical devices are considered in this analysis. At first, we formulate the reflection problem for the short end. The incident wave is assumed to be in the 0th mode (EG mode), but many reflected modes may occur at the shorted plane (see Fig. 3).

We expand the electric field

$$E_z = \underbrace{\Psi_0(x)\exp(-\gamma_0 y)}_{\text{incident wave}} + \underbrace{\sum_{n=0}^{\infty} A_n \Phi_n(x)\exp(\gamma_n y)}_{\text{reflected wave}} \quad (27)$$

where A_n is the expansion coefficient of the nth mode.

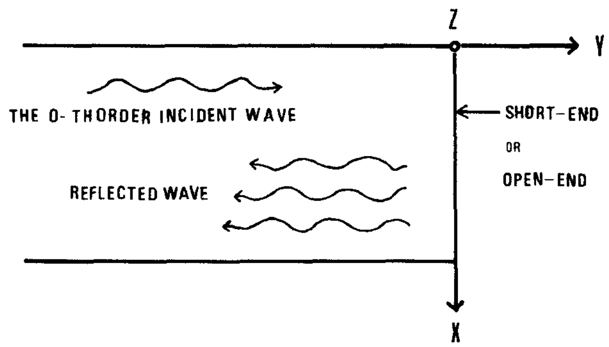


Fig. 3. Terminations of a ferrite stripline.

From the short boundary condition at $y = 0$ we have

$$\Psi_0(x) + \sum_{n=0}^{\infty} A_n \Phi_n(x) = 0. \tag{28}$$

To approximate (28) with a finite summation for n, say up to N, A_n is determined to minimize the squared error Λ_N as in (29); that is, to determine A_n satisfying (30)

$$\Lambda_N = \int_0^a \left| \Psi_0(x) + \sum_{n=0}^{N} A_n \Phi_n(x) \right|^2 dx \tag{29}$$

$$\frac{\partial \Lambda_N}{\partial A_n^*} = 0, \qquad \text{for any } n = 0,1,\cdots,N. \tag{30}$$

When the N is increased up to infinite, (28) will be satisfied exactly. In another expression, A_n is a solution to the linear equations

$$[C][A] = [D] \tag{31}$$

$$C_{i,j} = \int_0^a \Phi_i^*(x) \cdot \Phi_j(x)\, dx, \qquad i,j = 0,1,\cdots,N \tag{32}$$

$$D_i = -\int_0^a \Phi_i^*(x) \cdot \Psi_0(x)\, dx, \qquad i = 0,1,\cdots,N \tag{33}$$

$$[A]^t = (A_0, A_1, \cdots, A_N). \tag{34}$$

Such a formulation of solving a matrix equation is suitable for computer analysis. The detailed expressions for (32) and (33) are given in Appendix 2.

It should be noticed that there is another method of determining the mode-expansion coefficients A_n; for example, the point-matching method. A study of the numerical calculation itself is necessary to shorten the calculation time.

Next we will similarly solve the reflection problem for the open-end; that is, the magnetic field should be expanded by eigenmodes.

$$H_x = \eta_0(x) \exp(-\gamma_0 y) + \sum_{n=0}^{\infty} B_n \cdot \xi_n(x) \exp(\gamma_n y). \tag{35}$$

From the open-boundary condition at the $y = 0$ plane, we have

$$\eta_0(x) + \sum_{n=0}^{\infty} B_n \cdot \xi_n(x) = 0. \tag{36}$$

The detailed expressions of coefficient matrix and column vector to determine the expansion coefficients B_n are given in Appendix 2.

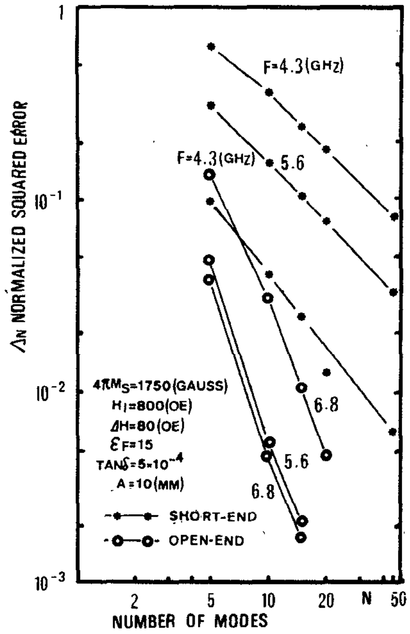


Fig. 4. Normalized squared error Λ_N.

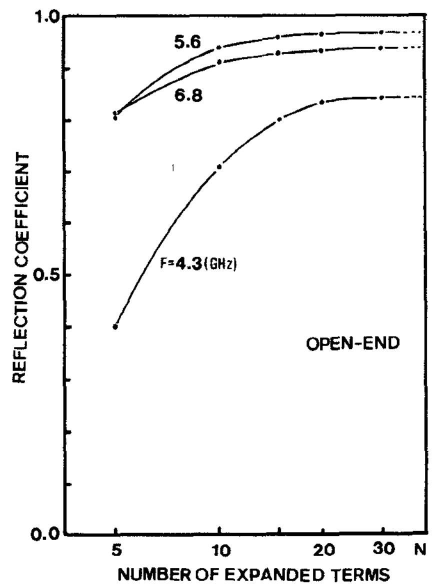


Fig. 5. Convergence of a calculated reflection coefficient.

IV. Calculated Results

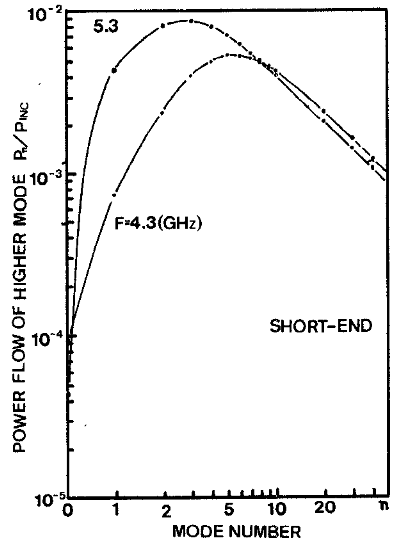
The reflection problems of the short end and the open end were numerically solved by the aid of the mode expansion method described in Section III.

Generally speaking, the convergence for the short end is slower than that for the open end. The convergences of the squared error defined by (29) are shown in Fig. 4.

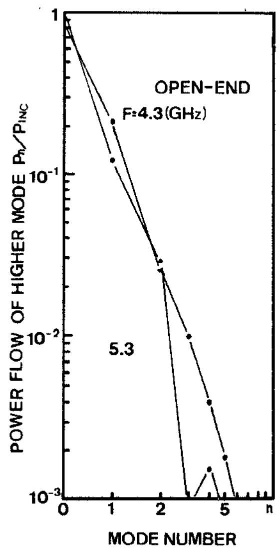
$$\Lambda_N \sim N^{-1} \qquad \text{(for short end)}$$

$$\Lambda_N \sim N^{-3} \qquad \text{(for open end).}$$

Fig. 5 shows the convergence of the reflection coefficient for the 0th mode.



(a)



(b)

Fig. 6. Power flow of higher modes. (a) Short end. (b) Open end.

TABLE II
THE CROSS TERMS OF THE EIGENMODES IN POWER FLOW

Total reflection coefficient	The cross terms are neglected	The cross terms are considered
for the "open-end"	1.0809	1.0022
for the "short-end"	0.8606	0.9979

at f=7.4(GHz)

In the frequencies where $\mu_{eff} < 0$, all modes except the 0th mode (EG mode) are evanescent, and so these modes are highly absorbed with a small amount of ferrite loss, and they cannot return to the input port.

The power flow of each mode P_n at $y = 0$, are plotted in Fig. 6. Fig. 6 shows that for the short end, higher order modes are more strongly generated at the reflection plane.

As stated in Section II, there is no superpositionality in power flow and, therefore, if the power flow of all modes are added, the total value reflected will not be equal to that of incident wave. But if the cross terms of the different modes are accounted in the power flow, this inconsistency does not occur. An example of numerical results is given in Table II.

The transmission characteristics depend on the line structure as well as the dispersion of ferrite medium, of which the latter is approximately given, as follows [5]:

$$\mu = 1 + \frac{(\omega_i + j\omega\alpha) \cdot \omega_m}{(\omega_i + j\omega\alpha)^2 - \omega^2} \tag{37}$$

$$\kappa = \frac{\omega \cdot \omega_m}{(\omega_i + j\omega\alpha)^2 - \omega^2} \tag{38}$$

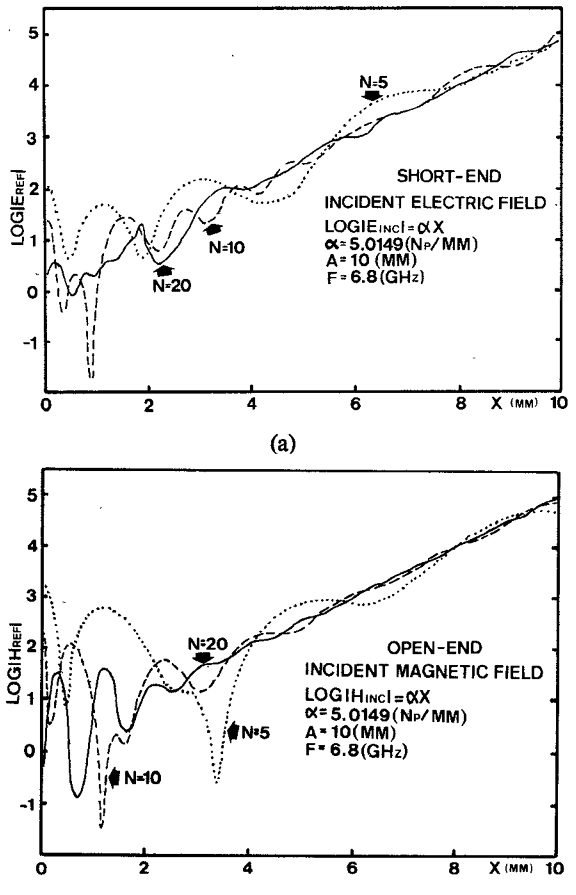


Fig. 7. Calculated field distribution of a reflected wave. (a) Short end. (b) Open end.

where

$$\omega_i = -\gamma H_i$$

$$\omega_m = -\gamma 4\pi M_s$$

and γ, H_i, and $4\pi M_s$ are the gyromagnetic ratio, an internal magnetic field, and a saturation magnetization, respectively.

Therefore, the frequency characteristics of the reflection coefficient may be calculated using (37), (38). Calculations were performed for the range of 4.0–8.0 GHz. As regards the effective permeability of ferrite $\mu_{\text{eff}} < 0$, the frequency concerned ranges from f_1 to f_2, as given below:

$$f_1 = \sqrt{f_i(f_i + f_m)} \quad (39)$$

$$f_2 = f_i + f_m. \quad (40)$$

For our case, $f_1 = 4.0$ GHz and $f_2 = 7.14$ GHz, and in such a frequency range the difference between the reflections from a short end and an open end is very large, about 40 dB, as shown in Fig. 8.

Fig. 7 shows the calculated field distribution of the backward wave at the $y = 0$ plane with N as a parameter, where N eigenmodes are used to expand the reflected wave. According to the increase of N, the calculated field distribution of the backward wave converges to that of the incident wave.

Some important features in Fig. 8 are: 1) the reduction

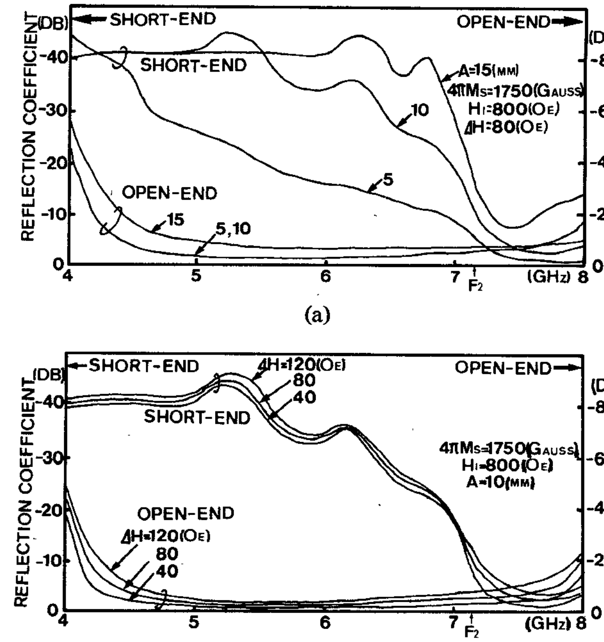


Fig. 8. Calculated reflection coefficients.

of level difference between for a short end and for an open end beyond f_2; 2) a large level difference for a wide stripline width; and 3) small reflection for large ΔH, where ΔH means the linewidth of gyromagnetic resonance.

V. Experimental Results

Experiments were performed by using YIG slab, with $4\pi M_s = 1750$ G, $\Delta H = 83$ Oe, and thickness $d = 1$ mm. The reflection coefficients were measured for the frequency range from 4 to 8 GHz by means of a network analyzer.

Experimental results are shown in Fig. 9. The ripples in the reflected power are found because of imperfection in the transition from coaxial line to stripline. The measured level differences of reflected power of the short end and the open end vary from 15 dB to above 25 dB. These values can be accepted to show a good resemblance to the calculated ones, if it is remembered that level differences of above 20 dB can no more be measured in the case of the residual VSWR of 1.2 of the OSM connector.

From these theoretical and experimental investigations, it is clear that the reflection problems of the EG mode transmission line are quite different from those of the ordinal dielectric substrate transmission line.

These characteristics can be understood physically by another way of explanation, especially for the case of the open end.[1] The EG mode can propagate along the open edge and, in the case of the open end, all peripheries are open and so it can turn around to the input port along the open edge. In the case of the short end, the EG mode faces the short part and the energy will be absorbed through mode conversion [6], and so a small amount of the energy may come back to the input port.

[1] This was suggested by Dr. Hines.

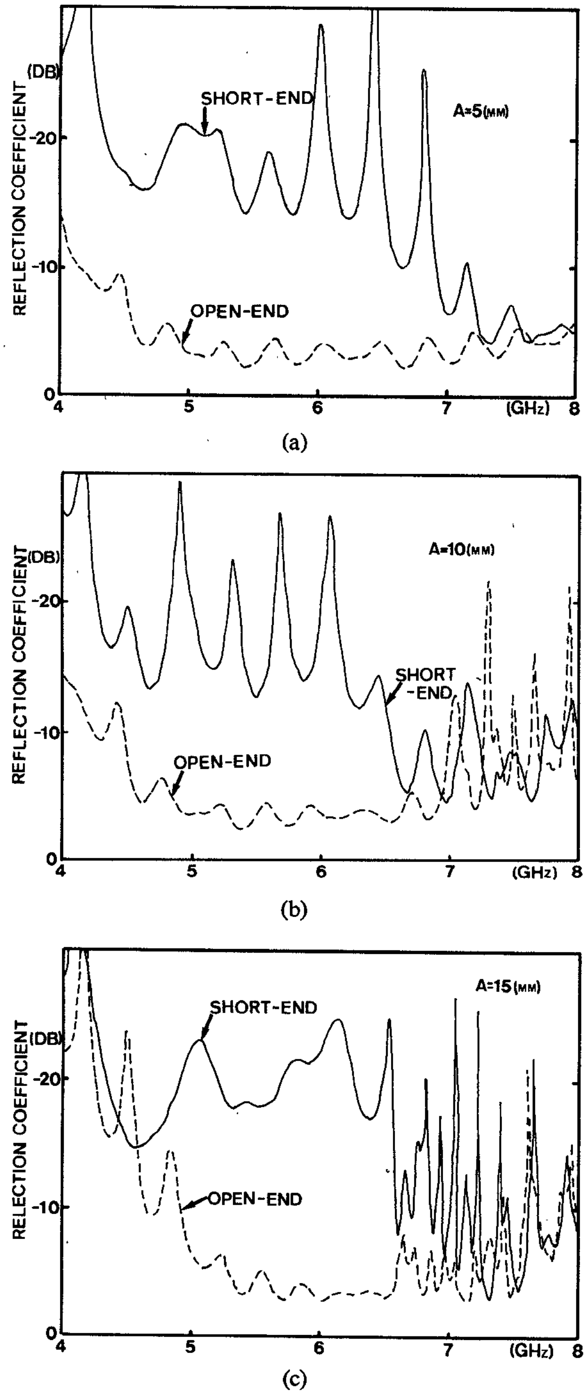


Fig. 9. Measured reflection coefficient. (a) $A = 5$ mm. (b) $A = 10$ mm. (c) $A = 15$ mm.

VI. A New Type of EG Mode Isolator

From these analytical and experimental investigations, it is shown that if the ferrite stripline terminates in the short end, reflections will be largely suppressed. This suggested a new type of EG mode isolator, as shown in Fig. 10, as an application of this fact. Experimental results of forward and backward transmissions are shown in Fig. 11. Insertion loss is about 2.5 dB, isolation is about 35 dB, and a usable frequency range is about 0.6 GHz. The characteristics of this isolator are not as good as the new type of EG mode isolator formerly described by us [3].

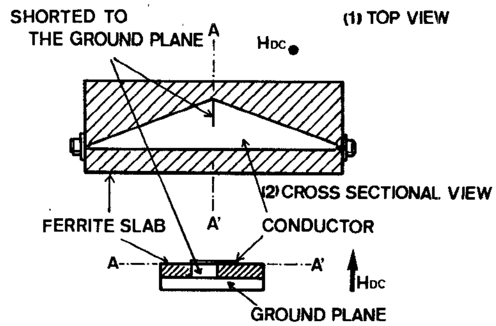


Fig. 10. A new type of EG mode isolator.

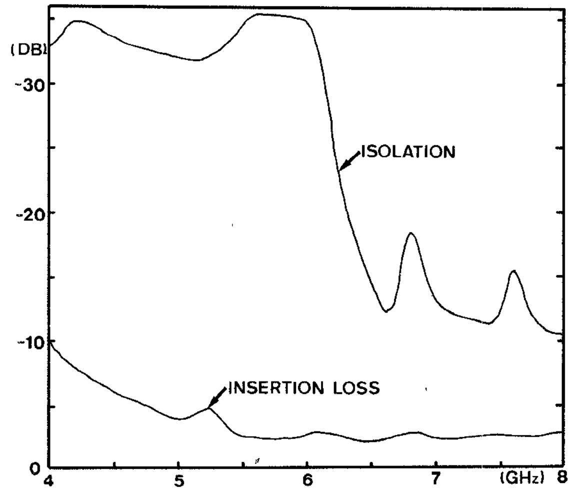


Fig. 11. Characteristics of a new type of EG mode isolator.

VII. Conclusion

The transmission characteristics of stripline on the ferrite substrate are investigated, and the features of eigenmode function are clarified by comparing them with those of an ordinary stripline. The reflection problems of a short end and an open end are treated and analyzed by means of an eigenmode expansion method. Calculated values of a reflected power show characteristics similar to experimental ones.

It is shown that for the case of a short end, these reflections are extremely small (about −40 dB), and as an application of this fact, a new type of EG mode isolator was proposed, built, and tested.

Appendix 1
Superpositionality of Power Flow of Eigenmodes

$$P_{n,m} = \mathrm{Re}\left\{\frac{1}{j\omega\mu_{\mathrm{eff}}}\cdot\left[A_n^* A_m \exp\left[-(\gamma_n^* + \gamma_m)y\right]\int_0^a \Psi_n^* \cdot \left(\gamma_m\Psi_m + j\frac{\kappa}{\mu}\Psi_m'\right)dx + A_n A_m^* \exp\left[-(\gamma_n + \gamma_m^*)y\right]\int_0^a \Psi_m^* \cdot \left(\gamma_n\Psi_n + j\frac{\kappa}{\mu}\Psi_n'\right)dx\right]\right\}.$$

$$P_{n,m} = \mathrm{Re}\left\{\frac{1}{j\omega\mu_{\mathrm{eff}}} \cdot \left[A_n^* A_m \exp\left[-(\gamma_n^* + \gamma_m)y\right]\gamma_m \int_0^a \Psi_n^*\Psi_m \, dx + A_n A_m^* \exp\left[-(\gamma_n + \gamma_m^*)y\right]\gamma_m^* \int_0^a \Psi_n \Psi_m^* \, dx + j\frac{\kappa}{\mu}\cdot A_n^* A_m \exp\left[-(\gamma_n^* + \gamma_m)y\right] \int_0^a \Psi_n^* \Psi_m' \, dx - j\frac{\kappa}{\mu} A_n A_m^* \exp\left[-(\gamma_n + \gamma_m^*)y\right] \int_0^a \Psi_n \Psi_m^{*\prime} \, dx\right]\right\} = \mathrm{Re}\left\{\frac{1}{j\omega\mu_{\mathrm{eff}}}[\text{real number}]\right\} = 0. \tag{A1}$$

Equation (23) is used in manipulating (A1), and it is assumed that $\mu = \mu^*$ and $\kappa = \kappa^*$.

Appendix 2
The Detailed Expressions for (32) and (33)

By its definition, it is seen that $[C]$ is a Hermite matrix, and so we write down $C_{n,m}$ only for $n \leq m$.

$$C_{0,0} = \frac{\exp(2\,\mathrm{Re}\{\alpha\}a) - 1}{2\,\mathrm{Re}\{\alpha\}} \tag{A2}$$

$$C_{0,m} = \frac{\exp(\alpha^* a)(-1)^m - 1}{\alpha^{*2} + \Gamma_m^2}\left(\alpha^* + j\frac{\kappa}{\mu}\gamma_m\right) \tag{A3}$$

$$C_{n,n} = \left(1 + \left|\frac{\kappa}{\mu}\cdot\gamma_n\right|^2 \Big/ \Gamma_n^2\right)\frac{a}{2} \tag{A4}$$

$$C_{n,m} = \frac{2j}{\Gamma_n^2 - \Gamma_m^2}\left(\frac{\kappa}{\mu}\cdot\Gamma_n + \frac{\kappa^*}{\mu^*}\cdot\Gamma_m^*\right), \quad \text{for } n - m = \text{odd} \tag{A5}$$

or

$$C_{n,m} = 0, \quad \text{for } n - m = \text{even}.$$

The elements of column vector $[D]$ are given as follows:

$$D_0 = \frac{\exp(-2j\,\mathrm{Im}\{\alpha\}a) - 1}{2j\,\mathrm{Im}\{\alpha\}} \tag{A6}$$

$$D_n = \frac{\exp(-\alpha a)(-1)^n - 1}{\alpha^2 + \Gamma_n^2}\left(\alpha + j\frac{\kappa^*}{\mu^*}\cdot\gamma_n^*\right). \tag{A7}$$

Similarly, for the open end, $[C]$ and $[D]$ are given as follows:

$$C_{0,0} = \left|\frac{\varepsilon}{\mu}\right| \cdot \frac{\exp(2\,\mathrm{Re}\{\alpha\}a) - 1}{2\,\mathrm{Re}\{\alpha\}} \tag{A8}$$

$$C_{0,m} = \sqrt{\frac{\varepsilon^*}{\mu^*}} \cdot \frac{\exp(\alpha^* a)(-1)^m - 1}{\alpha^{*2} + \Gamma_m^2}\left(\frac{\gamma_m}{j\omega\mu}\alpha^* - \frac{\kappa}{\mu}\omega\varepsilon\right) \tag{A9}$$

$$C_{n,n} = \left(\left|\frac{\gamma_n}{\omega\mu}\right|^2 + \left|\frac{\kappa}{\mu}\frac{\omega\varepsilon}{\Gamma_n}\right|^2\right)\frac{a}{2} \tag{A10}$$

$$C_{n,m} = -\frac{2j}{\Gamma_n^2 - \Gamma_m^2}\left(\frac{\gamma_n^*}{\omega\mu^*}\frac{\kappa}{\mu}\omega\varepsilon + \frac{\gamma_m}{\omega\mu}\frac{\kappa^*}{\mu^*}\omega\varepsilon^*\right), \quad \text{for } n - m = \text{odd} \tag{A11}$$

or

$$C_{n,m} = 0 \quad \text{for } n - m = \text{even}$$

$$D_0 = \left|\frac{\varepsilon}{\mu}\right| \cdot \frac{\exp(-2j\,\mathrm{Im}\{\alpha\}a) - 1}{-2j\,\mathrm{Im}\{\alpha\}} \tag{A12}$$

$$D_n = \frac{\exp(-\alpha a)(-1)^n - 1}{\alpha^2 + \Gamma_n^2}\left(\frac{\gamma_n^*}{j\omega\mu} - \frac{\kappa^*}{\mu^*}\omega\varepsilon^*\right)\sqrt{\frac{\varepsilon}{\mu}}. \tag{A13}$$

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References

[1] M. E. Hines, "Reciprocal and non-reciprocal modes of propagation in ferrite stripline and microstrip devices," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-19, pp. 442–451, May 1972.
[2] T. J. Gerson and J. S. Nadan, "Surface electromagnetic modes of a ferrite slab," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-22, pp. 757–763, August 1974.
[3] K. Araki, T. Koyama, and Y. Naito, "A new type of isolator using the edge-guided mode," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-23 (Letter), p. 321, March 1975.
[4] K. Kurokawa, *Introduction to the Theory of Microwave Circuits.* New York: Academic, 1969.
[5] R. F. Soohoo, *Theory and Application of Ferrites.* Englewood Cliffs, NJ: Prentice-Hall, 1960.
[6] K. Araki, T. Koyama, and Y. Naito, "New edge guided mode devices," *IEEE MTT-S Symposium, Digest of Papers*, pp. 250–253, May 1975.